On (1,2)*-Semi-Generalized-Star Homeomorphisms

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Abstract: *The aim of this paper is to introduce the concept of (1,2)*-semi-generalised-star closed sets (briefly (1,2)*-sg*-closed sets) and study some of its properties. Their corresponding pre-(1,2)*-sg*-closed maps and (1,2)*-sg*-irresolute maps are defined and studied in this paper.*

Keywords: *(1,2)*-sg*-closed set, (1,2)*-sg*-open set, pre-(1,2)*-sg*-closed map, (1,2)*-sg*-irresolute map.*

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1. Introduction

The study of bitopological spaces was first initiated by Kelly [2] in the year 1963. Recently Ravi, Lellis Thivagar, Ekici and Many others [3, 5 - 14] have defined different weak forms of the topological notions, namely, semi-open, preopen, regular open and α-open sets in bitopological spaces.

In this paper, we introduce the notion of (1,2)*-semi-generalized-star closed (briefly, (1,2)*-sg*-closed) sets and investigate their properties. By using the class of (1,2)*-sg*-closed sets, we study the properties of (1,2)*-sg*-open sets, pre-(1,2)*-sg*-closed maps and (1,2)*-sg*-irresolute maps. In most of the occasions our ideas are illustrated and substantiated by some suitable examples.

2. Preliminaries

Throughout this paper, X and Y denote bitopological spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) respectively, on which no separation axioms are assumed.

Definition 2.1

Let S be a subset of X. Then S is called $\tau_{1,2}$-open [13] if S = A ∪ B, where $A \in \tau_1$ and $B \in \tau_2$.

The complement of $\tau_{1,2}$-open set is called $\tau_{1,2}$-closed.

The family of all $\tau_{1,2}$-open (resp. $\tau_{1,2}$-closed) sets of X is denoted by (1,2)*-O(X) (resp. (1,2)*-C(X)).

Example 2.2

Let X = {a, b, c}, τ_1 = {φ, X, {b}} and τ_2 = {φ, X, {c}}. Then the sets in {φ, X, {b}, {c}, {b, c}} are $\tau_{1,2}$-open and the sets in {φ, X, {a}, {a, b}, {a, c}} are $\tau_{1,2}$-closed.

Definition 2.3

Let A be a subset of a bitopological space X. Then

(i) the $\tau_{1,2}$-closure of A [12], denoted by $\tau_{1,2}$-cl(A), is defined by ∩ {U: A ⊆ U and U is $\tau_{1,2}$-closed };

(ii) the $\tau_{1,2}$-interior of A [12], denoted by $\tau_{1,2}$-int(A), is defined by ∪ {U: U ⊆ A and U is $\tau_{1,2}$-open}.

Remark 2.4

Notice that $\tau_{1,2}$-open subsets of X need not necessarily form a topology.

Now we recall some definitions and results, which are used in this paper.

Definition 2.5

A subset S of a bitopological space X is said to be (1,2)*-semi-open [12] if S ⊆ $\tau_{1,2}$-cl($\tau_{1,2}$-int(S)).

The complement of (1,2)*-semi- open set is called (1,2)*-semi-closed.

The family of all (1,2)*-semi-open sets of X will be denoted by (1,2)*-SO(X).

The (1,2)*-semi-closure of a subset S of X is, denoted by (1,2)*-scl(S), defined as the intersection of all (1,2)*-semi-closed sets containing S.

Definition 2.6

A subset S of a bitopological space X is said to be a (1,2)*-sg-closed [10] if (1,2)*-scl(S) ⊂ U whenever S ⊂ U and U ∈ (1,2)*-SO(X).

Definition 2.7

A subset S of a bitopological space X is said to be a (1,2)*-g-closed [14] if $\tau_{1,2}$-cl(S) ⊂ U whenever S ⊂ U and U ∈ (1,2)*-O(X).

The complement of (1,2)*-g-closed set is (1,2)*-g-open.

Definition 2.8

A map f : X → Y is called

(i) (1,2)*-continuous [12] if $f^{-1}(V)$ is $\tau_{1,2}$-closed in X for every $\sigma_{1,2}$-closed set V in Y.

(ii) (1,2)*-open map [11] if the image of every $\tau_{1,2}$-open set is an $\sigma_{1,2}$-open.

(iii) (1,2)*-irresolute [9] if the inverse image of (1,2)*-semi-open set is (1,2)*- semi-open.

Definition 2.9

A map f : X → Y is called (1,2)*-homeomorphism [11] if f is bijection, (1,2)*-continuous and (1,2)*-open.

3. (1,2)*-Semi-Generalised- Star-Closed Sets

Definition 3.1

A subset A of a bitopological space X is called a (1,2)*-semi-generalised-star-closed (briefly, (1,2)*-sg*-closed) if $\tau_{1,2}$-cl(A) ⊂ U whenever A ⊂ U and U is (1,2)*-semi-open in X.

Example 3.2

Let X = {a, b, c}, τ_1 = {φ, X, {b}} and τ_2 = {φ, X, {c}}. Then the sets in {φ, X, {b}, {c}, {b, c}} are $\tau_{1,2}$-open. Clearly the sets in {φ, X, {a}, {a, b}, {a, c}} are (1,2)*-sg*-closed.

Theorem 3.3

Every $\tau_{1,2}$-closed set is (1,2)*-sg*-closed.

Proof:

Let A be a $\tau_{1,2}$-closed subset of X. Let A ⊂ U and U be (1,2)*-semi-open. $\tau_{1,2}$-cl(A) = A, since A is $\tau_{1,2}$-closed. Therefore $\tau_{1,2}$-cl(A) ⊂ U. Hence A is (1,2)*-sg*-closed.

Remark 3.4

The converse of Theorem 3.3 need not be true as shown in the following example.

Example 3.5

Let X = {a, b, c}, τ_1 = {φ, X, {a, b}} and τ_2 = {φ, X}. Then the set {a, c} is (1,2)*-sg*-closed but not $\tau_{1,2}$-closed.

Theorem 3.6

Every (1,2)*-sg*-closed set is (1,2)*-g-closed .

Proof:

Let A be a (1,2)*-sg*-closed subset of X. Let A ⊂ U and U be $\tau_{1,2}$-open. Then U is (1,2)*-semi-open since every $\tau_{1,2}$-open set is (1,2)*-semi-open. Since A is (1,2)*-sg*-closed and U is (1,2)*-semi-open, we have $\tau_{1,2}$-cl(A) ⊂ U. Hence A is (1,2)*-g-closed .

Remark 3.7

The converse of Theorem 3.6 need not be true as shown in the following example.

Example 3.8

Let X = {a, b, c, d}, τ_1 = {φ, X, {a, b}} and τ_2 = {φ, X, {b, c}, {a, b, c}}. Then the sets in {φ, X, {a, b}, {b, c}, {a, b, c}} are $\tau_{1,2}$-open. Then the set {b, d} is (1,2)*-g-closed but not (1,2)*-sg*-closed.

Remark 3.9

(1,2)*-semi-closed sets and (1,2)*-sg*-closed sets are independent of each other.

Example 3.10

Let X = {a, b, c}, τ_1 = {φ, X, {a}, {b}, {a, b}} and τ_2 = {φ, X, {a, c}}. Then the sets in {φ, X, {a}, {b}, {a, b}, {a, c}} are $\tau_{1,2}$-open. Clearly the set {b} is (1,2)*-semi-closed set but not (1,2)*-sg*-closed.

Example 3.11

Let X = {a, b, c, d}, τ_1 = {φ, X, {a, b}} and τ_2 = {φ, X, {b, c}, {a, b, c}}. Then the sets in {φ, X, {a, b}, {b, c}, {a, b, c}} are $\tau_{1,2}$-open. Clearly the set {a, c, d} is (1,2)*-sg*-closed set but not (1,2)*-semi-closed.

Remark 3.12

Union of two (1,2)*-sg*-closed sets need not be a (1,2)*-sg*-closed.

Example 3.13

Let X = {a, b, c}, τ_1 = {φ, X, {a}, {c}, {a, c}, {b, c}} and τ_2 = {φ, X, {a}, {a, b}}. Then the sets in {φ, X, {a}, {c}, {a, b}, {a, c}, {b, c}} are $\tau_{1,2}$-open and the sets in {φ, X, {a}, {b}, {c}, {a, b}, {b, c}} are (1,2)*-sg*-closed. But {a} ∪ {c} = {a, c} is not (1,2)*-sg*-closed.

Theorem 3.14

A (1,2)*-sg*-closed set which is (1,2)*-semi-open is $\tau_{1,2}$-closed.

Proof:

Let A be a (1,2)*-sg*-closed set which is (1,2)*-semi-open. We have $A \subset A$ and A is (1,2)*-semi-open. Since A is (1,2)*-sg*-closed, $\tau_{1,2}$-cl(A) ⊂ A. It is well known that $A \subset \tau_{1,2}$-cl(A). Hence A is $\tau_{1,2}$-closed.

Result 3.15

Being (1,2)*-semi-open is a sufficient condition for a (1,2)*-sg*-closed set to be $\tau_{1,2}$-closed. However this condition is not necessary. There are (1,2)*-sg*-closed sets which are $\tau_{1,2}$-closed but not (1,2)*-semi-open.

Example 3.16

Let X = {a, b, c}, τ_1 = {φ, X, {a}, {c}, {a, c}} and τ_2 = {φ, X, {c}}. Then the sets in {φ, X, {a}, {c}, {a, c}} are $\tau_{1,2}$-open.Clearly the set {b} is (1,2)*-sg*-closed set and $\tau_{1,2}$-closed but not (1,2)*-semi-open.

Theorem 3.17

If A is (1,2)*-sg*-closed and $A \subset B \subset \tau_{1,2}$-cl(A), then B is (1,2)*-sg*-closed.

Proof:

Let A be a (1,2)*-sg*-closed subset of X. Since $A \subset B \subset \tau_{1,2}$-cl(A), we have $\tau_{1,2}$-cl(B) $\subset \tau_{1,2}$-cl(A). Let $B \subset U$ and U be (1,2)*-semi-open. Then $A \subset U$, $\tau_{1,2}$-cl(A) ⊂ U since A is (1,2)*-sg*-closed. Hence $\tau_{1,2}$-cl(B) ⊂ U. Hence B is (1,2)*-sg*-closed.

Theorem 3.18

Let A be (1,2)*-sg*-closed in X but not $\tau_{1,2}$-closed. Then for every $\tau_{1,2}$-open set $U \subset A$, there exists an $\tau_{1,2}$-open set V such that A intersects V and $\tau_{1,2}$-cl(U) does not intersect V.

Proof:

Assume that A is (1,2)*-sg*-closed but not $\tau_{1,2}$-closed. Let $U \subset A$ and U be $\tau_{1,2}$-open. We claim that $A \not\subset \tau_{1,2}$-cl(U). If $A \subset \tau_{1,2}$-cl(U), then $U \subset A \subset \tau_{1,2}$-cl(U) and U is $\tau_{1,2}$-open. Hence A is (1,2)*-semi-open. Therefore A is (1,2)*-sg*-closed and (1,2)*-semi-open which implies A is $\tau_{1,2}$-closed. But A is not $\tau_{1,2}$-closed. Hence $A \not\subset \tau_{1,2}$-cl(U). Hence there exists $x \in A$ and $x \notin \tau_{1,2}$-cl(U). Let V = $(\tau_{1,2}\text{-cl}(U))^c$. Then V is $\tau_{1,2}$-open and $\tau_{1,2}$-cl(U) does not intersect V. Since $x \notin \tau_{1,2}$-cl(U), we have $x \in (\tau_{1,2}\text{-cl}(U))^c$. Hence $x \in V$. Since $x \in A$ and $x \in V$, $A \cap V \neq \varphi$, A intersects V and $\tau_{1,2}$-cl(U) does not intersect V.

Definition 3.19

Let X be a bitopological space and $A \subset X$. Then (1,2)*-frontier of A, denoted by (1,2)*-Fr(A), is defined to be the set $\tau_{1,2}$-cl(A) \ $\tau_{1,2}$-int(A).

Theorem 3.20

Let A be (1,2)*-sg*-closed and $A \subset U$ where U is $\tau_{1,2}$-open. Then (1,2)*-Fr(U) $\subset \tau_{1,2}$-int(A^c).

Proof:

Let A be (1,2)*-sg*-closed and let $A \subset U$ where U is $\tau_{1,2}$-open. Then $\tau_{1,2}$-cl(A) ⊂ U. Take any $x \in$ (1,2)*-Fr(U).We have $x \in \tau_{1,2}$-cl(U) \ $\tau_{1,2}$-int(U). Hence $x \in \tau_{1,2}$-cl(U) \ U since U is $\tau_{1,2}$-open. Hence $x \notin U$. Therefore $x \notin \tau_{1,2}$-cl(A). Hence $x \in (\tau_{1,2}\text{-cl}(A))^c$. Therefore $x \in \tau_{1,2}$-int(A^c). Hence (1,2)*-Fr(U) $\subset \tau_{1,2}$-int(A^c).

Definition 3.21

A bitopological space X is called RM-space if every subset in X is either $\tau_{1,2}$-open or $\tau_{1,2}$-closed.

Theorem 3.22

In a RM-space X, every (1,2)*-sg*-closed set is $\tau_{1,2}$-closed.

Proof:

Let X be a RM-space. Let A be a (1,2)*-sg*-closed subset of X. Then A is $\tau_{1,2}$-open or $\tau_{1,2}$-closed. If A is $\tau_{1,2}$-closed, then nothing to prove. If A is $\tau_{1,2}$-open, then A is (1,2)*-semi-open. Since A is (1,2)*-sg*-closed and A is (1,2)*-semi-open, by Theorem 3.14, A is $\tau_{1,2}$-closed.

Remark 3.23

The converse of Theorem 3.22 need not be true as shown in the following example.

Example 3.24

Let X = {a, b, c}, τ_1 = {φ, X, {a}} and τ_2 = {φ, X}. Then the sets in {φ, X, {a}} are $\tau_{1,2}$-open and the sets in {φ, X, {b, c}} are (1,2)*-sg*-closed. Therefore every (1,2)*-sg*-closed set is $\tau_{1,2}$-closed. But X is not a RM-space.

Definition 3.25

A subset A of a bitopological space X is said to be (1,2)*-semi-generalised-star-open (briefly, (1,2)*-sg*-open) if A^c is (1,2)*-sg*-closed.

Theorem 3.26

Every $\tau_{1,2}$-open set is (1,2)*-sg*-open.

Proof:

Let A be an $\tau_{1,2}$-open set of X. Then A^c is $\tau_{1,2}$-closed. Also A^c is (1,2)*-sg*-closed since every $\tau_{1,2}$-closed set is (1,2)*-sg*-closed. Hence A is (1,2)*-sg*-open.

Remark 3.27

The converse of Theorem 3.26 need not be true as shown in the following example.

Example 3.28

Let X = {a, b, c, d}, τ_1 = {φ, X, {a, b, d}} and τ_2 = {φ, X, {b, c, d}}. Then the sets in {φ, X, {a, b, d}, {b, c, d}} are $\tau_{1,2}$-open. Clearly the set {b} is (1,2)*-sg*-open but not $\tau_{1,2}$-open.

Theorem 3.29

Every (1,2)*-sg*-open set is (1,2)*-g-open.

Proof:

Let A be a (1,2)*-sg*-open set of X. Then A^c is (1,2)*-sg*-closed. Also A^c is (1,2)*-g-closed, since every (1,2)*-sg*-closed set is (1,2)*-g-closed . Hence A is (1,2)*-g-open.

Remark 3.30

The converse of Theorem 3.29 need not be true as shown in the following example.

Example 3.31

Let X = {a, b, c}, τ_1 = {φ, X, {a}} and τ_2 = {φ, X, {a, c}}. Then the sets in {φ, X, {a}, {a, c}} are $\tau_{1,2}$-open. Then the set {c} is (1,2)*-g-open but not (1,2)*-sg*-open.

Remark 3.32

Intersection of two (1,2)*-sg*-open sets need not be a (1,2)*-sg*-open.

Example 3.33

Let X = {a, b, c}, τ_1 = {φ, X, {a}, {c}, {a, c}, {b, c}} and τ_2 = {φ, X, {a}, {a, b}}. Then the sets in {φ, X, {a}, {c}, {a, b}, {a, c}, {b, c}} are $\tau_{1,2}$-open and the sets in {φ, X, {a}, {c}, {a, b}, {a, c}, {b, c}} are (1,2)*-sg*-open. But {a, b} ∩ {b, c} = {b} is not (1,2)*-sg*-open.

Theorem 3.34

If A is (1,2)*-sg*-open and $\tau_{1,2}$-int(A) ⊂ B ⊂ A. Then B is (1,2)*-sg*-open.

Proof:

Let A be (1,2)*-sg*-open. Hence A^c is (1,2)*-sg*-closed. Since $\tau_{1,2}$-int(A) ⊂ B ⊂ A, $(\tau_{1,2}\text{-int}A)^c \supset B^c \supset A^c$. Therefore $A^c \subset B^c \subset \tau_{1,2}$-cl($A^c$). Hence by Theorem 3.17, B^c is (1,2)*-sg*-closed. Hence B is (1,2)*-sg*-open.

Theorem 3.35

If A is (1,2)*-sg*-open and A ⊃ F where F is $\tau_{1,2}$-closed then (1,2)*-Fr(F) ⊂ $\tau_{1,2}$-int(A).

Proof:

Given that A is (1,2)*-sg*-open and A ⊃ F where F is $\tau_{1,2}$-closed. Then A^c is (1,2)*-sg*-closed, $A^c \subset F^c$ and F^c is $\tau_{1,2}$-open. By Theorem 3.20 (1,2)*-Fr(F^c) ⊂ $\tau_{1,2}$-int(A). Hence (1,2)*-Fr(F) ⊂ $\tau_{1,2}$-int(A) since (1,2)*-Fr(F^c) = (1,2)*-Fr(F).

Theorem 3.36

In a RM-space X, every (1,2)*-sg*-open set is $\tau_{1,2}$-open.

Proof:

Let X be a RM-space. Let A be a (1,2)*-sg*-open subset of X. Then A^c is (1,2)*-sg*-closed. Since X is a RM-space, A^c is $\tau_{1,2}$-closed. Hence A is $\tau_{1,2}$-open.

Remark 3.37

The converse of Theorem 3.36 need not be true as shown in the following example.

Example 3.38

Let X = {a, b, c}, τ_1 = {φ, X, {b}} and τ_2 = {φ, X}. Then the sets in {φ, X, {b}} are $\tau_{1,2}$-open and the sets in {φ, X, {b}} are (1,2)*-sg*-open. Therefore every (1,2)*-sg*-open set is $\tau_{1,2}$-open. But X is not a RM-space.

Theorem 3.39

Any singleton set is either (1,2)*-semi-closed or (1,2)*-sg*-open.

Proof:

Take {x}, if it is (1,2)*-semi-closed then nothing to prove. If it is not (1,2)*-semi-closed, then $\{x\}^c$ is not (1,2)*-semi-open. Therefore X is the only (1,2)*-semi-open set containing $\{x\}^c$. Therefore $\{x\}^c$ is (1,2)*-sg*-closed. Hence {x} is (1,2)*-sg*-open. Therefore {x} is (1,2)*-semi-closed or (1,2)*-sg*-open.

4.(1,2)*-Semi-Generalised-Star Homeomorphisms

Definition 4.1

A function f: X → Y is called a (1,2)*-closed if f(V) ∈ (1,2)*-C(Y) for every $\tau_{1,2}$-closed set V in X.

Theorem 4.2

A function f : X → Y is (1,2)*-closed if and only if $\sigma_{1,2}$-cl[f(A)] ⊆ f [$\tau_{1,2}$-cl(A)] for every A ⊆ X.

Proof:

Let f be (1,2)*-closed and let A ⊆ X. Then f [$\tau_{1,2}$-cl(A)] ∈ (1,2)*-C(Y). But f(A) ⊆ f [$\tau_{1,2}$-cl(A)]. Then $\sigma_{1,2}$-cl[f(A)] ⊆ f [$\tau_{1,2}$-cl(A)]. Conversely, let A ⊆ X be a $\tau_{1,2}$-closed set. Then by assumption, $\sigma_{1,2}$-cl[f(A)] ⊆ f [$\tau_{1,2}$-cl(A)] = f(A). This shows that f(A) ∈ (1,2)*-C(Y). Hence f is (1,2)*-closed.

Definition 4.3

A function f: X → Y is called a (1,2)*-sg*-continuous if f^{-1}(V) is (1,2)*-sg*-closed in X for every $\sigma_{1,2}$-closed set V of Y.

Theorem 4.4

Let f: X → Y be a (1,2)*-homeomorphism. Then a subset A is (1,2)*-sg*-closed in Y ⇒ f^{-1} (A) is (1,2)*-sg*-closed in X.

Proof:

Let f: X → Y be a (1,2)*-homeomorphism. Let A be a (1,2)*-sg*-closed subset of Y. Let B = f^{-1} (A). Now to prove that B is (1,2)*-sg*-closed in X. Let U be any (1,2)*-semi-open set with B ⊂ U. Then f(B) ⊂ f(U). Therefore f(f^{-1} (A)) ⊂ f(U). Since f is (1,2)*-bijective, f(f^{-1} (A)) = A. Therefore A ⊂ f(U). We claim that f(U) is (1,2)*-semi-open. Since U is (1,2)*-semi-open, U ⊂ $\tau_{1,2}$-cl($\tau_{1,2}$-int(U)). Then f(U) ⊂ f($\tau_{1,2}$-cl($\tau_{1,2}$-int(U))) ⊂ $\sigma_{1,2}$-cl(f($\tau_{1,2}$-int(U))), since f is (1,2)*-continuous and f(U) ⊂ $\sigma_{1,2}$-cl($\sigma_{1,2}$-int f(U)) since f is (1,2)*-open. Therefore f(U) is (1,2)*-semi-open. Since A ⊂ f(U), f(U) is (1,2)*-semi-open and A is (1,2)*-sg*-closed. Therefore $\sigma_{1,2}$-cl(A) ⊂ f(U). Hence f^{-1}($\sigma_{1,2}$-cl(A)) ⊂ f^{-1} (f(U)). Since f is a (1,2)*-homeomorphism, f^{-1}($\sigma_{1,2}$-cl(A)) = $\tau_{1,2}$-cl(f^{-1} (A)). Therefore $\tau_{1,2}$-cl (f^{-1}(A)) ⊂ f^{-1}(f(U)). Therefore $\tau_{1,2}$-cl(B) ⊂ U. It means B is (1,2)*-sg*-closed in X. Hence f^{-1} (A) is (1,2)*-sg*-closed.

Theorem 4.5

Let f: X → Y be a (1,2)*-homeomorphism. A subset A is (1,2)*-sg*-open in Y ⇒ f^{-1} (A) is (1,2)*-sg*-open in X.

Proof:

A is (1,2)*-sg*-open in Y ⇒ A^c is (1,2)*-sg*-closed in Y ⇒ f^{-1} (A^c) is (1,2)*-sg*-closed in X ⇒ $[f^{-1}(A)]^c$ is (1,2)*-sg*-closed in X ⇒ f^{-1} (A) is (1,2)*-sg*-open in X.

Theorem 4.6

Let f: X → Y be a (1,2)*-homeomorphism. A subset A is (1,2)*-sg*-closed in X ⇒ f(A) is (1,2)*-sg*-closed in Y.

Proof:

Let f: X → Y be a (1,2)*-homeomorphism. Assume that A is (1,2)*-sg*-closed in X. Let B = f(A). Now to prove that B is (1,2)*-sg*-closed in Y. Let U be a (1,2)*-semi-open set with B ⊂ U. That is f(A) ⊂ U. Hence f^{-1} (f(A)) ⊂ f^{-1} (U). Since f is (1,2)*-bijective, f^{-1}(f(A)) = A. Therefore A ⊂ f^{-1} (U). Since U is (1,2)*-semi-open and f is a (1,2)*-homeomorphism, f^{-1} (U) is (1,2)*-semi-open, we have A ⊂ f^{-1}(U), f^{-1} (U) is (1,2)*-semi-open and A is (1,2)*-sg*-closed. Therefore $\tau_{1,2}$-cl(A) ⊂ f^{-1} (U). Hence f($\tau_{1,2}$-cl(A)) ⊂ f(f^{-1} (U)). Since f is a (1,2)*-closed map, $\sigma_{1,2}$-cl(f(A)) ⊂ f($\tau_{1,2}$-cl(A)). Therefore $\sigma_{1,2}$-cl(f(A)) ⊂ f[f^{-1} (U)]. Hence $\sigma_{1,2}$-cl(B) ⊂ U. It means B is (1,2)*-sg*-closed in Y. Therefore image of a (1,2)*-sg*-closed set is (1,2)*-sg*-closed.

Theorem 4.7

Let f: X → Y be a (1,2)*-homeomorphism. A is (1,2)*-sg*-open in X ⇒ f(A) is (1,2)*-sg*-open in Y.

Proof:

A is (1,2)*-sg*-open in X ⇒ A^c is (1,2)*-sg*-closed in X.
⇒ f(A^c) is (1,2)*-sg*-closed in Y.
⇒ $[f(A)]^c$ is (1,2)*-sg*-closed in Y.
⇒f(A) is (1,2)*-sg*-open in Y.

Definition 4.8

Let X and Y be two bitopological spaces. A map f: X → Y is called a pre (1,2)*-sg*-closed if for each (1,2)*-sg*-closed set A in X, f(A) is (1,2)*-sg*-closed in Y.

Theorem 4.9

Every (1,2)*-homeomorphism is a pre (1,2)*-sg*-closed map.

Proof :

It follows from Theorem 4.6.

Definition 4.10

Let X and Y be two bitopological spaces. A map f : X → Y is called a pre- (1,2)*-sg*-open if for each (1,2)*-sg*-open set A in X, f(A) is (1,2)*-sg*-open in Y.

Theorem 4.11

Every (1,2)*-homeomorphism is a pre-(1,2)*-sg*-open map.

Proof:

It follows from Theorem 4.7.

Definition 4.12

Let X and Y be two bitopological spaces. A map f : X → Y is called (1,2)*-sg*-irresolute if for each (1,2)*-sg*-closed set A in Y, f^{-1} (A) is (1,2)*-sg*-closed in X.

Theorem 4.13

Every (1,2)*-homeomorphism is (1,2)*-sg*-irresolute map.

Proof:

It follows from Theorem 4.4.

Theorem 4.14

f: X → Y is (1,2)*-sg*-irresolute if and only if inverse image of every (1,2)*-sg*-open set in Y is (1,2)*-sg*-open in X.

Proof:

A is (1,2)*-sg*-open in Y ⇔ A^c is (1,2)*-sg*-closed in Y
⇔ $f^{-1}(A^c)$ is (1,2)*-sg*-closed in X.
⇔ $[f^{-1}(A)]^c$ is (1,2)*-sg*-closed in X.
⇔ f^{-1} (A) is (1,2)*-sg*-open in X.

Definition 4.15

Let X and Y be two bitopological spaces. A map f : X → Y is called a (1,2)*-sg*-homeomorphism if f is (1,2)*-bijective, f is (1,2)*-sg*-irresolute and f^{-1} is (1,2)*-sg*-irresolute.

Theorem 4.16

If f: X → Y is (1,2)*-bijective, then the following are equivalent.

1. f is (1,2)*-sg*-irresolute and f is pre-(1,2)*-sg*-closed.
2. f is (1,2)*-sg*-irresolute and f is pre-(1,2)*-sg*-open.
3. f is (1,2)*-sg*-homeomorphism.

Proof:

1 ⇒ 2. We have f: X → Y is (1,2)*-bijective, f is (1,2)*-sg*- irresolute and f is pre-(1,2)*-sg*-closed. Since f is pre-(1,2)*-sg*-closed, A is (1,2)*-sg*-open in X ⇒ A^c is (1,2)*-sg*-closed in X.

⇒ $f(A^c)$ is (1,2)*-sg*-closed in Y.

⇒ $[f(A)]^c$ is (1,2)*-sg*-closed in Y.

⇒ f(A) is (1,2)*-sg*-open in Y.

Hence f is a pre-(1,2)*-sg*-open map.

2 ⇒ 3. We have f: X → Y is (1,2)*-bijective, f is (1,2)*-sg*-irresolute and f is pre-(1,2)*-sg*-open. Since f is pre-(1,2)*-sg*-open, A is (1,2)*-sg*-open in X ⇒ f(A) is (1,2)*-sg*-open in Y ⇒ $(f^{-1})^{-1}(A)$ is (1,2)*-sg*-open in Y.

Hence f^{-1} is (1,2)*-sg*-irresolute. Hence f is a (1,2)*-sg*-homeomorphism.

3 ⇒ 1. We have f: X → Y is (1,2)*-bijective, f is (1,2)*-sg*-irresolute and f^{-1} is (1,2)*-sg*-irresolute. Now f^{-1} is (1,2)*-sg*-irresolute ⇒ f is pre-(1,2)*-sg*-closed.

Definition 4.17

Let X and Y be two bitopological spaces. A map f : X → Y is called (1,2)*-sg*-closed map if for each $\tau_{1,2}$-closed set F of X, f(F) is (1,2)*-sg*-closed in Y.

Definition 4.18

Let X and Y be two bitopological spaces. A map f: X → Y is called (1,2)*-sg*-open if for each $\tau_{1,2}$-open set U of X, f(U) is (1,2)*-sg*-open in Y.

Theorem 4.19

Every (1,2)*-homeomorphism is a (1,2)*-sg*-homeomorphism.

Proof:

It follows from the fact that every (1,2)*-continuous map is (1,2)*-sg*-continuous [10] and every (1,2)*-open map is a (1,2)*-sg*-open map [11].

Remark 4.20

The converse of Theorem 4.19 need not be true as shown in the following example.

Example 4.21

Let X = {a, b, c} = Y, τ_1 = {φ, X, {a}, {a, b}} and τ_2 = {φ, X, {b}}. Then the sets in {φ, X, {a}, {b}, {a, b}} are $\tau_{1,2}$-

open. Let $\sigma_1 = \{\varphi, Y, \{a\}\}$ and $\sigma_2 = \{\varphi, Y, \{a, b\}\}$. Then the sets in $\{\varphi, Y, \{a\}, \{a, b\}\}$ are $\sigma_{1,2}$-open. Define $f : X \to Y$ by $f(a) = a$, $f(b) = b$ and $f(c) = c$. Clearly f is not a (1,2)*-homeomorphism since f is not an (1,2)*-open map. However f is (1,2)*-sg*-homeomorphism.

Theorem 4.22

If $f : X \to Y$ is an (1,2)*-irresolute (1,2)*-closed map, then F is (1,2)*-sg*-closed in X $\Rightarrow$ f(F) is (1,2)*-sg*-closed in Y.

Proof:

Let F be a (1,2)*-sg*-closed subset of X. Now to prove that f(F) is (1,2)*-sg*-closed in Y. Let $f(F) \subset U$ and U be (1,2)*-semi-open. Then $F \subset f^{-1}(U)$. Since U is (1,2)*-semi-open and f is (1,2)*-irresolute. Therefore $f^{-1}(U)$ is (1,2)*-semi-open. Since F is (1,2)*-sg*-closed, $F \subset f^{-1}(U)$ and $f^{-1}(U)$ is (1,2)*-semi-open, $\tau_{1,2}\text{-cl}(F) \subset f^{-1}(U)$. Hence $f(\tau_{1,2}\text{-cl}(F)) \subset f(f^{-1}(U)) \subset U$. Since f is a (1,2)*-closed map, $\sigma_{1,2}\text{-cl}(f(F)) \subset f(\tau_{1,2}\text{-cl}(F))$. Hence $\sigma_{1,2}\text{-cl}(f(F)) \subset U$. Therefore f(F) is (1,2)*-sg*-closed.

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